Calculating the volume of the tunnels

Key Learning Area	Unit or lesson title and main focus questions	Most appropriate level and suggested number of lessons
Mathematics	**Calculating the volume of the tunnels** How much rock material has to be excavated to build the rail tunnels?	**Stage 5** 1–2 lessons

Teacher briefing

In seeking answers to interesting calculations, students begin to understand how engineers apply mathematical knowledge to solving 'real-world' problems. When discussing the Sydney Metro Northwest tunnels, for example, many people might ask the following questions. How much rock material has to be excavated to make the rail tunnels? How heavy is all that rock? Students will enjoy finding answers to these questions and sharing them with peers.

Assessment

The teacher may assess both the correct working of the problems and students' appreciation of the application of Mathematics to finding 'real-world' solutions.

Key terms and vocabulary

Cylinder, volume, density, mass.

Syllabus links

Mathematics K–10

(MA5.2–2WM) interprets mathematical or real-life situations, systematically applying appropriate strategies to solve problems.

Background information

To minimise environmental change, allow for multiple land use and provide direct travel between points, trains travelling along the Sydney Metro Northwest alignment will pass through tunnels between Epping and Bella Vista. Each will be 15 kilometres long and have an internal diameter of 6 metres.

Construction of the tunnels is a major operation because it involves the drilling and removal of large quantities of rock. The question many people ask is: How much rock material has to be removed to make these rail tunnels?

The rock through which the tunnels will pass is mostly quartz-rich sandstone, which has a low porosity. This means that there are few spaces between particles that make up the rock. An estimate of the density of the sandstone rock is 2.35 tonne per cubic metre (Tm^{-3}). Some parts of the tunnel will be composed of rock that is slightly more or less dense, but this density value is a reasonable average.

Figure 15: Hills Showground tunnel and Castle Hill crossover tunnel, May 2016.

Learning experiences

Calculating the volume of rock that has to be removed

The tunnels between Epping and Bella Vista are simply long cylinders lying on their side. The length of each cylinder (h) is 15,000 metres. The diameter of each tunnel cylinder is 6 metres. Their radius (r) is therefore 3 metres.

The formula for the volume of a cylinder is the length multiplied by the cross sectional area of the tunnel. The volume (V) is calculated using the formula:

$$V = \pi r^2 h$$

Determine the volume of rock that has to be removed to make the tunnels. Remember there are two tunnels. Show your working.

Answer: 438,252.2 m^3

Calculating the mass of rock to be excavated from the tunnel

The density of a substance is the mass of the particles making up a certain volume of the substance. The density of water is one tonne per cubic metre. The density of rock depends on the composition of particles making up the rock and whether the rock has air spaces between the particles that compose it.

Density is determined using the formula D = M / V where D is the density expressed in tonne per m^3, M is the mass expressed in tonnes and V is the volume expressed in m^3. This algorithm can be rearranged to make M the subject of the formula. Attempt to do this, clearly stating what process you used to obtain your rearranged algorithm.

D = Density

M = Mass

V = Volume

D = M / V

M = D x V

V = M / D

Answer: M=DV

Using the rearranged algorithm, the mass of rock material that will have to be removed from the twin Sydney Metro Northwest tunnels can then be calculated.

Answer: 1,007,980 tonnes

Important note: The volumes and measurements outlined here are based on data from the Sydney Metro Northwest 's *Environmental Impact Statement 2* in 2012. More detailed engineering work has been undertaken since then following the awarding of the major tunnelling construction contract in June, 2013.

Teacher references and extension work

When designing tunnels such as those between Epping and Bella Vista, an engineer needs to carry out many calculations as shown on page 165.

Students may like to discuss some of the further complexities involved as shown in Figure 16 (below).

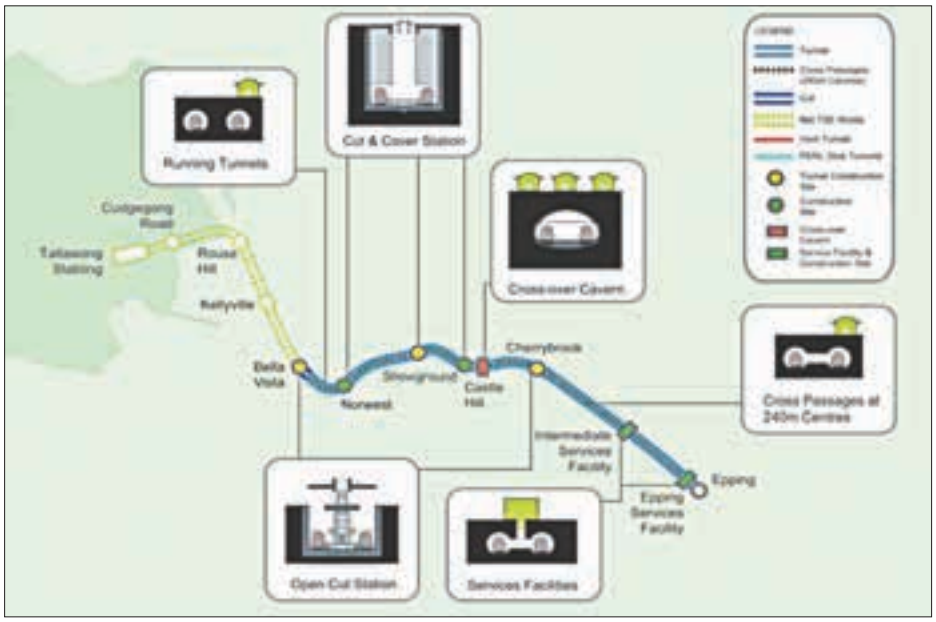

Figure 16: Diagram of the underground components of the Epping–Bella Vista Tunnel segment of Sydney Metro Northwest.

For example, while the twin tunnels are approximately 6 m in diameter, some other sections of tunnel may be larger – but the precise sizes and shapes of these are to be determined. The road header machines will excavate these other sections, which include caverns and TBM launch chambers as well as areas for mechanical and electrical services. The tunnels also have cross passages every 240 metres. In addition there are underground stations.

Students may discuss what additional percentage of volume they would add to accommodate some of these options.

Figure 17: A modern railway tunnel in construction. See more at: **https://www.sydneymetro.info/tunnelling**

Figure 18: Norwest underground station.

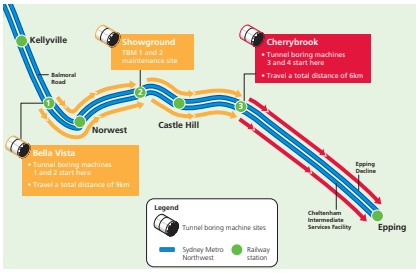

Figure 19: How the Sydney Metro Northwest tunnels were created.